Refined Softened Truss Model with Efficient Solution Procedure for Prestressed Concrete Membranes

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Abstract: This article presents a refinement of the rotating-angle softened truss model (RA-STM) for prestressed concrete (PC) membrane elements. To refine the RA-STM, an efficient solution procedure to solve the set of nonlinear equations is presented. Appropriate average stress-strain relationships for concrete and steel bars are also used in the solution procedure. The theoretical results obtained from the refined RA-STM are compared with some experimental results found in the literature and also with the original RA-STM. It is shown that the refined RA-STM captures better the behavior of PC membrane elements under shear and also predicts better the shear strength. DOI: 10.1061/(ASCE)ST.1943-541X.0002044. © 2018 American Society of Civil Engineers.

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Introduction

Based on the original truss models proposed in the beginning of the last century (Ritter 1899; Mörsch 1902), several analytical models have been developed to compute and understand the behavior of reinforced concrete (RC) membrane elements. In these models, the idealized resistance mechanism is a plain truss model, which includes concrete to resist compressive stresses and reinforcement (steel bars) to resist tensile stresses. Such models are also useful to analyze complex structures primarily subjected to in-plane stresses (e.g., shear walls, box beams, and containment structures). Such structures can be analyzed as the association of smaller two-dimensional (2D) elements subjected to in-plane stresses, that is, membrane elements.

Among these analytical models, the softened truss model (STM) proposed by Hsu (1988) has been widely accepted and used by researchers. As a novelty, this model incorporates a smeared stress ($\sigma$)-strain ($\varepsilon$) relationship for concrete in compression in the struts that accounts for the softening effect previously reported by Robinson and Demorieux (1972), instead of simpler $\sigma$–$\varepsilon$ relationship from uniaxial compression tests with concrete samples.

Based on the experimental results from several controlled tests with RC panels, carried out on in the Universal Panel Tester at the University of Houston (Hsu et al. 1995a, b), the STM has been improved over the years.

In order to account for that, after cracking, principal stresses rotate as a result of the internal redistribution of stresses (Hsu 1993), the angle of the principal compressive stresses was allowed to vary as the load increases. Because of this, the model was renamed as rotating-angle softened truss model (RA-STM) (Pang and Hsu 1995).

New smeared $\sigma$–$\varepsilon$ relationships for concrete in compression (Belarbi and Hsu 1995; Zhang and Hsu 1998) and for steel bars in tension including the stiffening effect (Belarbi and Hsu 1994) and the dowel action (Pang and Hsu 1995) has been proposed. Additionally, a smeared $\sigma$–$\varepsilon$ relationship for concrete in tension is also incorporated to properly account for the stiffening effect and also for the contribution of the tensile concrete in the early stages (Belarbi and Hsu 1994, 1995; Pang and Hsu 1995).

The RA-STM assumes that the reference frame to write the equilibrium and compatibility equations coincides with the rotating principal directions of internal stresses in concrete. For this reason, the contribution of concrete in shear is not considered. However, such small contribution actually exists because of the interlocking of the aggregates along the cracks and also because the actual direction of cracks is not exactly equal to the direction of the principal compressive stresses (Pang and Hsu 1992). In order to incorporate the shear ($\tau$)-shear strain ($\gamma$) relationship for concrete, Pang and Hsu (1996) and Hsu and Zhang (1997) proposed the fixed-angle softened truss model (FA-STM). In this model, the reference frame is at a fixed angle, equal to the angle of the principal directions of the external stresses applied to the RC member. After cracking, this angle no longer coincides with the rotation angle of the principal direction of internal stresses. Because equilibrium and compatibility equations are written in this fixed reference frame, the $\tau$–$\gamma$ relationship for concrete must be considered.

Finally, according to Hsu and Mo (2010), Poisson’s effect for the cracked state should also be incorporated in order to RA-STM and FA-STM better predict the postpeak behavior of the RC membranes. Based on experimental results, Hsu and Zhu (2002) defined the so-called Hsu-Zhu ratios and proposed the softened membrane model (SMM).

Nowadays, the application of prestress is current in many RC structures under plane stress states. Prestress induces compressive stresses, which, combined with the in-plane stresses due to loading,
result in a biaxial stress (shear + compression) that delays the cracking of concrete and increases the shear resistance. Hence, the RA-STM was extended for prestressed concrete (PC) membrane elements (Hsu 1993). Later, Laskar et al. (2007) extended the SMM for PC membrane elements and proposed the so-called softened membrane model for prestressed concrete (SMM-PC).

For all the models previously mentioned, the calculation procedures to compute the behavior of RC and PC membrane elements are based on trial-and-error techniques, which may require a large calculation effort and also lose efficiency because of the large number of estimated parameters. For this reason, alternative solution procedures are need.

In recent studies the RA-STM, which is analytically simpler than FA-STM and SMM, was refined to compute the behavior of RC membrane elements (Silva and Horowitz 2015; Silva 2016). In another recent study, the combined-action softened truss model (CA-STM) was also refined (Silva et al. 2017). For this, alternative efficient solution procedures were proposed. The problem is formulated as a system of nonlinear equations with constraints, solved efficiently by using optimization algorithms. This avoids using trial-and-error techniques. Moreover, appropriate smeared $\sigma - \varepsilon$ relationships for concrete and steel bars were also used. Based on comparative analysis with experimental results, it was shown that the refined RA-STM captures the behavior well, including the postpeak behavior, of RC membrane elements.

In this paper, the RA-STM for PC membrane elements is refined using a similar methodology as for the refined RA-STM for RC membrane elements. The changes in the RA-STM formulation and the new efficient solution procedure are presented. The predictions from the theoretical model are compared with some experimental results of PC panels under shear. It is found that the refined RA-STM do well at capturing the behavior of such elements, including the shear resistance, with less calculation effort.

Refined RA-STM for PC Membrane Elements

This section summarizes the RA-STM for RC membrane elements (Pang and Hsu 1995; Belarbi and Hsu 1994, 1995), extended for PC membrane elements (Hsu and Mo 2010), and presents the changes that lead to the refined RA-STM with efficient solution procedure. The version of the RA-STM used in this study neglects the contribution of the concrete tensile strength, as well as the dowel action in the steel bars. Previous studies show that without these contributions, RA-STM do well at capturing the global behavior of RC membrane elements under shear, namely for high loading stages (Hsu 1993; Zhang and Hsu 1998; Pang and Hsu 1995; Silva and Horowitz 2015; Silva 2016).

Equilibrium Equations

Fig. 1 shows a PC membrane element (with prestress in the longitudinal and transverse direction) loaded with in-plane stresses. The $L$-$T$ and $D$-$R$ coordinate systems coincide with the direction of the longitudinal and transverse steel bars and with the principal direction of stresses in concrete, respectively. The in-plane stresses ($\sigma_L$, $\sigma_T$, and $\tau_{LT}$) in the PC membrane element are described as the sum of the following components: stresses in the concrete element ($\sigma_{L}^{c}$, $\sigma_{T}^{c}$, and $\tau_{LT}^{c}$), stresses in the non-prestress steel bars ($\rho_{fL}$ and $\rho_{fT}$), and stresses in the prestress steel bars ($\rho_{LPfL}$ and $\rho_{TPfT}$). The stresses in the steel bars are obtained from the reinforcement ratios ($\rho_{L}$, $\rho_{T}$, $\rho_{LP}$, and $\rho_{TP}$) and from the corresponding stresses ($f_{L}$, $f_{T}$, $f_{LP}$, and $f_{TP}$). From Fig. 1 the following three equilibrium equations are written:

$$
\begin{bmatrix}
\sigma_{L} \\
\sigma_{T} \\
\tau_{LT}
\end{bmatrix}
= 
\begin{bmatrix}
\sigma_{L}^{c} \\
\sigma_{T}^{c} \\
\tau_{LT}^{c}
\end{bmatrix}
+ 
\begin{bmatrix}
\rho_{fL} \\
\rho_{fT} \\
0
\end{bmatrix}
+ 
\begin{bmatrix}
\rho_{LPfL} \\
\rho_{TPfT} \\
0
\end{bmatrix}
$$

From the Mohr’s circle, the stresses in the concrete element ($\sigma_{L}^{c}$, $\sigma_{T}^{c}$, and $\tau_{LT}^{c}$) can be computed from the principal stresses in concrete ($\sigma_{R}$ and $\sigma_{B}$) and from the variable angle $\alpha_B$ between the coordinate systems $L$-$T$ and $R$-$D$ (angle of principal direction of tensile stresses in concrete, Fig. 1). In this study, the complement of the variable angle, $\alpha_D$, is used because it coincides with the angle of cracks to the $L$-axis. From this, the following equations can be written to relate the stresses in the coordinate systems $L$-$T$ and $R$-$D$:

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Substituting Eq. (2) into Eq. (1), gives

\[
\begin{bmatrix}
\sigma_L \\
\sigma_T \\
\tau_{LT}
\end{bmatrix} =
\begin{bmatrix}
\cos^2(\alpha_D) & \sin^2(\alpha_D) & 2\sin(\alpha_D) \cos(\alpha_D) \\
\sin^2(\alpha_D) & \cos^2(\alpha_D) & -2\sin(\alpha_D) \cos(\alpha_D) \\
-\sin(\alpha_D) \cos(\alpha_D) & \sin(\alpha_D) \cos(\alpha_D) & \cos^2(\alpha_D) - \sin^2(\alpha_D)
\end{bmatrix}
\begin{bmatrix}
\sigma_D \\
\sigma_R \\
0
\end{bmatrix}
\] (2)

Because concrete tensile strength is neglected ($\sigma_R = 0$) the three previous equations are rewritten as

\[
\sigma_L = \sigma_D \cos^2(\alpha_D) + \rho_{LT\sigma} \sigma_L
\] (4)

\[
\sigma_T = \sigma_D \sin^2(\alpha_D) + \rho_{TT\sigma} \sigma_T
\] (5)

\[
\tau_{LT} = -\sigma_D \sin(\alpha_D) \cos(\alpha_D)
\] (6)

### Compatibility Equations

As for the stresses, from the Mohr’s circle for strains the following three compatibility equations can be written to relate the average strains in the coordinate systems $L-T$ and $R-D$:

\[
\begin{bmatrix}
\varepsilon_L \\
\varepsilon_T \\
\gamma_{LT}
\end{bmatrix} =
\begin{bmatrix}
\cos^2(\alpha_D) & \sin^2(\alpha_D) & 2\sin(\alpha_D) \cos(\alpha_D) \\
\sin^2(\alpha_D) & \cos^2(\alpha_D) & -2\sin(\alpha_D) \cos(\alpha_D) \\
-2\sin(\alpha_D) \cos(\alpha_D) & 2\sin(\alpha_D) \cos(\alpha_D) & 2\cos^2(\alpha_D) - 2\sin^2(\alpha_D)
\end{bmatrix}
\begin{bmatrix}
\varepsilon_D \\
\varepsilon_R \\
0
\end{bmatrix}
\] (7)

From Eq. (7), the following invariant equation can be written (Hsu and Mo 2010), which assumes an important role for the efficient solution procedure proposed in this study:

\[
\varepsilon_L + \varepsilon_T = \varepsilon_D + \varepsilon_R
\] (8)

The third line of Eq. (7) can also be rewritten in a simpler form

\[
\gamma_{LT} = 2(\varepsilon_R - \varepsilon_D) \sin(\alpha_D) \cos(\alpha_D)
\] (9)

### Stress-Strain Relationships for Materials

The same average nonlinear $\sigma - \varepsilon$ relationships for concrete in compression and for non-prestress steel bars in tension (Fig. 2) justified in a previous study (Bernardo et al. 2012) are used in this paper. In addition, a nonlinear $\sigma - \varepsilon$ relationship for prestress steel bars is also incorporated (Fig. 2).

### Concrete in Compression

To characterize the behavior or concrete in compression in the struts, an average nonlinear $\sigma - \varepsilon$ relationship is used, accounting for the softening effect through a softening coefficient that affects both stress and strain. This $\sigma - \varepsilon$ relationship (Eqs. (10) and (11)), initially proposed by Belarbi and Hsu (1995), was refined by Zhu et al. (2001) to characterize better the shape of the $\sigma - \varepsilon$ curve after the postpeak stress.

\[
\sigma_D = \zeta f \left[ 2 \left( \frac{\varepsilon_D}{\xi_0} \right) - \left( \frac{\varepsilon_D}{\xi_0} \right)^2 \right] \quad \text{for} \quad \frac{\varepsilon_D}{\xi_0} \leq 1
\] (10)

Fig. 2. Average $\sigma - \varepsilon$ relationships: (a) concrete in compression; (b) non-prestress steel bars in tension; (c) prestress steel bars in tension
\[ \sigma_D = \zeta f'_L \left[ 1 - \left( \frac{\varepsilon_D/\zeta \varepsilon_0}{(4/\zeta) - 1} \right)^2 \right] \quad \text{for } \frac{\varepsilon_D}{\zeta \varepsilon_0} > 1 \]  

In Eqs. (10) and (11), \( f'_L \) = uniaxial compressive strength of concrete (peak stress); \( \varepsilon_0 \) = strain corresponding to the peak stress; and \( \zeta \) = softening coefficient. This coefficient [Eqs. (12)–(14)] is calibrated for high-strength concrete and accounts for the ratio of the transverse to the longitudinal resistance force in the reinforcements, \( \eta \) (Belarbi and Hsu 1995). For this study, this ratio must be corrected to account for the resistance force of the transverse and longitudinal prestress reinforcement [Eq. (13)]. This is because after concrete decompression, pretress steel bars contribute as ordinary reinforcement for the strength.

For this study, the validity condition from Pang and Hsu (1995) for parameter \( \eta \) is also used: \( 0.4 < \eta < 2.5 \). This range, confirmed by Zhang and Hsu (1998), was proposed from the results of experimental tests on RC panels under shear. From Eq. (14), the condition \( 0.4 < \eta < 2.5 \) is equivalent to the following one: \( 0.4 < \eta' < 1.0 \).

In Eq. (13), \( f_{Ly} \) and \( f_{T_y} \) are the yielding stresses for the longitudinal and transverse non-prestress reinforcement, respectively, while \( f_{Lyp1.0} \) and \( f_{Typ1.0} \) are the proportional conventional limit stresses to 0.1% (for strain) for the longitudinal and transverse pre-stress reinforcement, respectively (Fig. 2). These latter correspond to conventional yielding stresses usually defined for prestress steel for which experimental \( \sigma - \varepsilon \) curves don’t show a yielding plateau.

\[ \zeta = \left( \frac{5.8}{\sqrt{f'_L}} \right) \left( \frac{1}{\varepsilon_0} \right) \left( \frac{1}{\sqrt{1 + \frac{4000}{\eta'}}} \right) \]  

\[ \eta = \frac{\rho f_{Ly} + \rho_{ff} f_{T yp1.0}}{\rho f_{Ly} + \rho_{ff} f_{T yp1.0}} \]  

\[ \eta' = \eta \quad \text{if } \eta < 1 \]  

\[ \eta' = \frac{1}{\eta} \quad \text{if } \eta > 1 \]  

Non-Prestress Steel Bars in Tension

For non-prestress steel bars in tension embedded in concrete, an average \( \sigma - \varepsilon \) relationship accounting for the tension stiffening effect must be used because the tensile stress in the reinforcement is neglected in this study. A simplified version of the nonlinear \( \sigma - \varepsilon \) relationship proposed by Belarbi and Hsu (1994) is used in this study, namely the bilinear \( \sigma - \varepsilon \) relationship proposed by Pang and Hsu (1995), shown in Eqs. (15)–(21) (Fig. 2).

\[ f_S = E_S \varepsilon_S \quad \text{for } \varepsilon_S \leq \varepsilon_y' \]  

\[ f_S = (0.91 - 2BN) f_{Sy} + (0.02 + 0.25BN) E_S \varepsilon_S \quad \text{for } \varepsilon_S > \varepsilon_y' \]  

\[ \varepsilon_y' = \frac{f_y'}{E_S} \]  

\[ f_y' = (0.93 - 2BN) f_{Sy} \]  

\[ BN = \frac{1}{\rho} \left( \frac{f_{ct}}{f_{Sy}} \right)^{1.5} \left( \rho \geq 0.15\% \right) \]  

\[ f_{ct} = 0.313 \sqrt{f'_L} \text{MPa} \]  

In Eqs. (15)–(20) (valid for both longitudinal and transverse non-prestress reinforcement), \( f_S \) and \( \varepsilon_S \) = average tensile stress and strain in the steel bars, respectively; \( f_{Sy} = \) yielding stress; \( E_S = \) Young’s modulus for steel; and \( f_{ct} = \) tensile strength of concrete.

### Prestress Steel Bars in Tension

The calculation of the behavior of PC membrane elements for pre-decompression stage is not relevant because the response is linear and the range for the deformations is very small. For this reason, it is assumed that the RA-STM only starts the calculation procedure after the concrete decompression, as also assumed in previous studies (for instance, Hsu and Mo 1985).

To account for the initial deformation due to prestress at concrete decompression, the strain in the prestress reinforcement, \( \varepsilon_p \), is calculated as follows:

\[ \varepsilon_p = \varepsilon_{p,dec} + \varepsilon_S \]  

\[ \varepsilon_{p,dec} = \varepsilon_{p,i} + \varepsilon_{s,i} \]  

In the previous equations, \( \varepsilon_{p,i} = \) initial tensile strain in the prestress reinforcement due to prestress; and \( \varepsilon_{s,i} = \) initial compressive strain in the non-prestress reinforcement due to prestress. This latter is equal, in modulus, to the strain in the non-prestress reinforcement, necessary to reach the decompression (\( \varepsilon_{dec, i} \)). At decompression, \( \varepsilon_S = 0 \).

From Hooke’s law and on the basis of an equivalent concrete cross section, the initial strains are

\[ \varepsilon_{p,i} = \frac{f_{p,i}}{E_p} \]  

\[ \varepsilon_{s,i} = \frac{A_p f_{p,i}}{A_S (E_S - E_c) + E_c (A_c - A_p)} \]  

In the previous equations, \( f_{p,i} = \) initial stress in the prestress reinforcement due to prestress; \( E_p = \) Young’s modulus for prestress steel; \( E_c = \) Young’s modulus for concrete; \( A_p \) and \( A_S \) = total area of prestress and non-prestress steel, respectively; and \( A_c \) = area of concrete cross section.

To compute the stress in the prestress reinforcement, from the strain, a \( \sigma - \varepsilon \) relationship is needed. For prestress steel bars embedded in concrete, no smeared \( \sigma - \varepsilon \) relationship was found in the literature. However, for such reinforcement, the differences between the uniaxial and the stiffened \( \sigma - \varepsilon \) relationships can be neglected. This is because the tensile strain variation along the prestress reinforcement due to the external loading is negligible when compared to the same one due to prestress (Hsu and Mo 2010). For this reason, a uniaxial \( \sigma - \varepsilon \) relationship is adopted for this study, namely the one proposed by Hsu (1991), as shown in Eqs. (25) and (26) (Fig. 2).

\[ f_p = E_p (\varepsilon_{p,dec} + \varepsilon_S) \quad \text{for } \varepsilon_p \leq \varepsilon_{p,0.71} \]  

\[ f_p = \frac{E_p (\varepsilon_{p,dec} + \varepsilon_S)}{1 + \left( \frac{E_p (\varepsilon_{p,dec} + \varepsilon_S)}{E_p (\varepsilon_{p,dec} + \varepsilon_S)} \right)^{1/R}} \quad \text{for } \varepsilon_p > \varepsilon_{p,0.71} \]  

In Eq. (26), \( E_p' \) is called the Young’s modulus of Ramberg-Osgood, and \( R \) is a geometrical parameter related with the shape of the nonlinear curve. Both parameters need to be evaluated experimentally from uniaxial tensile tests (Hsu and Mo 2010). For this study, the values from Laskar et al. (2007) were considered, \( E_p' = 209 \text{ GPa (1 GPa = 145 ksi)} \) and \( R = 5 \), because the refined
RA-STM will be checked with the experimental results from the same authors, as presented later. In Eq. (26), \( f_{pu} \) = tensile strength of the prestress steel; and \( \varepsilon_{p0.7}^{*} \) = strain in the prestress steel corresponding to 0.7\( f_{pu} \). In the Ramberg-Osgood \( \sigma - \varepsilon \) curve, this stress defines the upper limit of the linear and elastic range characterized by Eq. (25).

In this study, the uniaxial \( \sigma - \varepsilon \) relationship from Eqs. (25) and (26) gave rise to convergence problems. This is because throughout the solution procedure (presented later) the strain \( \varepsilon \) undergoes small changes until the solution point is reached. As a consequence, \( \varepsilon_{p} \) also undergoes small changes. This causes discontinuity between the linear [Eq. (25)] and nonlinear [Eq. (26)] \( \sigma - \varepsilon \) curves. To overcome this problem, Eq. (26) is corrected to always coincide with Eq. (25) for \( \varepsilon_{p} = \varepsilon_{p0.7}^{*} \). For this, the strain corresponding to 0.7\( f_{pu} \) is first computed for the linear \( (L) \) part of the \( \sigma - \varepsilon \) relationship by using Hooke’s law

\[
\varepsilon_{p0.7}^{*} = \frac{0.7f_{pu}}{E_{p}}
\]

Next, the same strain corresponding to 0.7\( f_{pu} \) is computed for the nonlinear \( (NL) \) part of the \( \sigma - \varepsilon \) relationship by using the nonlinear equation, Eq. (26)

\[
0.7f_{pu} = \frac{E_{p}^{\prime}\varepsilon_{NL}^{*}}{1 + \left(\frac{E_{p}^{\prime}}{f_{pu}}\right)^{n}} \rightarrow \varepsilon_{NL}^{*} = \frac{0.7f_{pu}}{E_{p}}
\]

Then, the difference between the previous strains, \( \Delta \varepsilon_{p} \), is computed

\[
\Delta \varepsilon_{p} = \varepsilon_{L}^{*} - \varepsilon_{NL}^{*}
\]

To maintain continuity between the linear and nonlinear \( \sigma - \varepsilon \) curves, the nonlinear curve is translated along the strain axis by \( \Delta \varepsilon_{p} \). This correction is acceptable because the values for \( \Delta \varepsilon_{p} \) are very small (with an order of magnitude of \( \approx 10^{-6} \) to \( 10^{-5} \) in this study). Eq. (26) is rewritten as

\[
f_{p} = \frac{E_{p}^{\prime}(\varepsilon_{dec} + \varepsilon_{s} - \Delta \varepsilon_{p})}{1 + \left(\frac{E_{p}^{\prime}}{f_{pu}}(\varepsilon_{s} - \Delta \varepsilon_{p})\right)^{n}} \text{ for } f_{p} > \varepsilon_{p0.7}^{*}
\]

**Proportional Loading**

It is assumed that the ratios between the applied stresses remain constant (proportional loading). This condition allows one to relate the applied stresses in the \( L-T \) coordinate system with the principal tensile stress in the PC member \( (\sigma_{1}) \) in the 1-2 coordinate system, which coincides with the direction of principal stresses according to the applied load. For this, the following longitudinal, transverse, and shear proportionality coefficients \( (m_{L}, m_{T}, \text{and } m_{LT}, \text{respectively}) \) are used (Fig. 3) (Hsu and Mo 2010):

\[
m_{L} = \frac{\sigma_{L}}{\sigma_{1}}
\]

\[
m_{T} = \frac{\sigma_{T}}{\sigma_{1}}
\]

\[
m_{LT} = \frac{\tau_{LT}}{\sigma_{1}}
\]

From Mohr’s circle for stresses, the following equation can be written to relate \( \sigma_{1} \) with the in-plane applied stresses in the \( L-T \) coordinate system \( (\sigma_{L}, \sigma_{T}, \text{and } \tau_{LT}) \):

\[
\sigma_{1} = \frac{\sigma_{L} + \sigma_{T}}{2} \pm \sqrt{\left(\frac{\sigma_{L} - \sigma_{T}}{2}\right)^{2} + \tau_{LT}^{2}}
\]

Substituting the proportionality coefficients from Eqs. (31)–(33) into Eqs. (4)–(6), gives

\[
m_{L}\sigma_{1} - \rho_{T}\sigma_{L} - \rho_{L}\sigma_{T} = \sigma_{D}\cos^{2}(\alpha_{D})
\]

\[
m_{T}\sigma_{1} - \rho_{T}\sigma_{T} - \rho_{T}\sigma_{L} = \sigma_{D}\sin^{2}(\alpha_{D})
\]

\[
m_{LT}\sigma_{1} = -\sigma_{D}\sin(\alpha_{D})\cos(\alpha_{D})
\]

As for the RA-STM for RC members (Hsu 1993), Eqs. (35)–(37) can be combined into a single one [Eq. (38)] and a set of equations can be written to compute \( \sigma_{1} \) [Eqs. (39)–(42)]

\[
(m_{L}\sigma_{1} - \rho_{T}\sigma_{L} - \rho_{L}\sigma_{T})(m_{T}\sigma_{1} - \rho_{T}\sigma_{T} - \rho_{T}\sigma_{L}) = (m_{LT}\sigma_{1})^{2}
\]

\[
\sigma_{1} = \frac{1}{2A'}\left(B' \pm \sqrt{B'^{2} - 4A'C'}\right)
\]

\[
A' = m_{L}m_{T} - m_{LT}^{2}
\]

\[
B' = m_{L}(\rho_{T}\sigma_{L} + \rho_{T}\sigma_{T}) + m_{T}(\rho_{L}\sigma_{T} + \rho_{L}\sigma_{L})
\]

\[
C' = (\rho_{T}\sigma_{T} + \rho_{T}\sigma_{L})(\rho_{L}\sigma_{L} + \rho_{L}\sigma_{T})
\]

**Efficient Solution Procedure**

**Simplified Model for the Initial Estimates**

A simple linear elastic truss model is used to compute the initial estimates for \( \varepsilon_{L} \) and \( \varepsilon_{T} \), namely the Mohr compatibility truss model (MCTM). The MCTM assumes linear elastic \( \sigma - \varepsilon \) relationships for the materials \( (\varepsilon_{L} \text{ and } \varepsilon_{T}) \) can be estimated from Hooke’s law) and also neglects the concrete tensile strength (Hsu and Mo 2010).
Because the calculation procedure starts at concrete decompression, the influence of prestress is only incorporated through the contribution of the longitudinal and transverse prestress reinforcement in the equilibrium equations of the MCTM.

Additional Equations

The angle $\alpha_D$ can be written as function of the strains in the $L$-$T$ and $R$-$D$ coordinate systems. This is done from the compatibility equations [Eq. (7)] and by using trigonometric identities, giving Eqs. (43) and (44), which substitute $\alpha_D$ in the equilibrium equations to provide numerical stability to the solution procedure. After the calculation of the point solution, $\alpha_D$ can be computed from Eq. (45), which results from the combination of Eqs. (31) and (32)

$$\sin^2(\alpha_D) = \frac{\varepsilon_L - \varepsilon_D}{\varepsilon_R - \varepsilon_D}$$

$$\cos^2(\alpha_D) = \frac{\varepsilon_R - \varepsilon_L}{\varepsilon_R - \varepsilon_D}$$

$$\alpha_D = \arctan\left(\frac{\varepsilon_L - \varepsilon_D}{\varepsilon_T - \varepsilon_D}\right)$$

Nonlinear Equations for the Initial Estimates

As for the RA-STM for RC membrane elements, Eq. (37) can be rewritten for $\sigma_D$ [Eq. (46)] and substituted into Eqs. (35) and (36), which can be rewritten for $f_L$ and $f_T$, respectively [Eqs. (47) and (48)]

$$\sigma_D = \frac{-m_L \sigma_1 \sin(\alpha_D) \cos(\alpha_D)}{m_L - m_T \cos(\alpha_D)}$$

$$f_L = \frac{-\sigma_D \cos^2(\alpha_D) + m_L \sigma_1 - \rho \sigma_L f_{LP}}{\rho_T}$$

$$f_T = \frac{-\sigma_D \sin^2(\alpha_D) + m_T \sigma_1 - \rho \sigma_T f_{TP}}{\rho_T}$$

From previous equations and by applying Hooke’s laws, the following equations are written:

$$\varepsilon_L = \frac{(m_L - m_T \cot(\alpha_D)) \sigma_1}{E_s \rho_L} + \frac{\rho \sigma_L f_{LP,i}}{E_s}$$

$$\varepsilon_T = \frac{(m_T - m_L \tan(\alpha_D)) \sigma_1}{E_s \rho_T} + \frac{\rho \sigma_T f_{TP,i}}{E_s}$$

$$\varepsilon_D = \frac{-m_L \sigma_1}{E_s \sin(\alpha_D) \cos(\alpha_D)}$$

In Eqs. (49) and (50), $f_{LP,i}$ and $f_{TP,i}$ = initial stresses in the longitudinal and transverse prestress reinforcement due to prestress, respectively. From Eq. (45), a nonlinear equation to compute the initial estimate of $\alpha_D$ can be derived [Eq. (52)], with the strains computed from Eqs. (49)–(51). In Eq. (52), $F_{MCTM}$ is called the residual function for MCTM. After $\alpha_D$ is estimated from Eq. (52), the strains are recalculated from Eqs. (49)–(51) and used as the initial estimates to compute the point solution from RA-STM, as presented subsequently

$$F_{MCTM} = \frac{\varepsilon_L - \varepsilon_D}{\varepsilon_T - \varepsilon_D} - \tan^2(\alpha_D) = 0$$

Nonlinear Equations for RA-STM

By adding Eqs. (35) and (36) with Eqs. (44) and (43), respectively, and arranging terms, the following system with two nonlinear equations is obtained to compute the strains $\varepsilon_L$ and $\varepsilon_T$. In Eq. (53), $F_{RA-STM}$ is called the residual function for RA-STM

$$F_{RA-STM} = \begin{bmatrix} \frac{\varepsilon_T - \varepsilon_D}{\varepsilon_R - \varepsilon_D} - m_L \sigma_1 + \rho \sigma_L f_{LP} \\ \frac{\varepsilon_L - \varepsilon_D}{\varepsilon_R - \varepsilon_D} - m_T \sigma_1 + \rho \sigma_T f_{TP} \end{bmatrix} = 0$$

Algorithm for the Solution Procedure

From the previous, the algorithm for the refined RA-STM for PC membrane elements is presented in Fig. 4. To start the calculation, the user must specify some data related with the material properties, initial loading, prestress, the increment for $\varepsilon_D$ between iterations ($\Delta \varepsilon_D$), and the maximum number of solution points to be calculated ($n_{max}$).

![Flowchart for refined RA-STM for PC membrane elements](https://example.com/flowchart)

**Fig. 4.** Flowchart for refined RA-STM for PC membrane elements
The efficient calculation procedure estimates the stresses and strains at concrete decomposition from MCTM for the initial loading stresses $\mu \sigma_L$, $\mu \sigma_T$, and $\mu \tau_{LT}$, with $\mu = 10^{-3}$ to ensure that the estimates are compatible with linearity. At this stage, the initial values $\varepsilon_{L,i}, \varepsilon_{T,i},$ and $\varepsilon_{LT,i}$ are calculated from the strains corresponding to $\sigma_L$, $\sigma_T$, and $\tau_{LT}$ [Eq. (52)] multiplied by $\mu$ (Fig. 4). After the initialization of the calculation procedure, the strains are multiplied by 1,000 in order to be rescaled and to avoid the risk of ill-conditioned matrices. From this initial calculation procedure, the first solution point is calculated. The other solution points are estimated from the RA-STM.

To compute the full behavior of PC membrane elements, $\varepsilon_D$ is varied from zero with small increments ($\Delta \varepsilon_D = 4 \times 10^{-6}$). For each step, the system of nonlinear equations for RA-STM [Eq. (53)] is solved and the solution point is imposed to be the initial point of the subsequent step.

The calculation procedure with the refined RA-STM is repeated until one of the following stopping criteria are reached: (1) either $\varepsilon_D$ reaches a specified ultimate strain for the concrete in compression ($\varepsilon_{D,\text{ult}} \geq \varepsilon_{c,u}$), or (2) the $k$ (index representing the number of solution points) reaches the specified maximum value ($k \geq n_{\text{max}}$).

In this study, the ultimate strain $\varepsilon_{c,u}$ is defined from NP EN 1992-1-1 [CEN 2010] procedures as function of the concrete compressive strength.

All the solution procedures were implemented in MATLAB. To solve the nonlinear equations, the function lsqnonlin from MATLAB optimization toolbox was used.

### Comparative Analyses

To validate the refined RA-STM with efficient solution procedure for PC membrane elements, the theoretical predictions are compared with some experimental results and also with the theoretical predictions from the original RA-STM extended for PC membrane elements by Hsu and Mo (2010). Note that Hsu and Mo (2010) didn’t compare the predictions from the original RA-STM with experimental results of PC membrane elements. In fact, they only presented some numerical examples.

In the literature, only two experimental studies with PC membrane elements were found: Marti and Meyboom (1992) and Laskar et al. (2007). As commented by Laskar et al. (2007), the lack of such studies are primarily related to the high cost of the testing equipment.

Laskar et al. (2007) tested five PC panels (TA series) under pure shear (a special case of proportional loading, with $\sigma_2 = -\sigma_1$) to study the shear behavior and five PC panels (TE series) under sequential loading to study the constitutive relationships for materials. Only the results for panels from TA series are used in this study. The panels were tested at the Universal Panel Tester in the University of Houston, which allowed control of the testing conditions (Hsu et al. 1995a, b). All panels have sizes 139.8 × 139.8 × 17.8 cm (1 cm = 0.394 in.) and were tested under symmetrical biaxial stresses in the 1-2 coordinate system with steel bars oriented at 45°. Hence, the panels were tested under pure shear in the $L-T$ coordinate system ($\sigma_L = \sigma_T = 0$). Prestress was only applied in the longitudinal direction and the primary variable studied was the reinforcement ratio in each direction.

Marti and Meyboom (1992) tested two PC panels (PP series) on the shell element tester at University of Toronto. All panels have sizes 162.6 × 162.6 × 28.7 cm (1 cm = 0.394 in.) and were tested with similar conditions as for the TA panels from Laskar et al. (2007). The variable studied was the amount of prestressing in the 45° direction (one of the panels has no prestress). Laskar et al. (2007) reported that some deficiencies exist in the study from Marti and Meyboom (1992). For instance, the tester was not equipped with a servocontrolled system, so the ultimate stage, such as the postpeak behavior, was not captured accurately. Despite of this, the experimental results from Marti and Meyboom (1992) are used in this study to check the ability of the refined RA-STM with efficient solution procedure to model the preultimate response.

Table 1 summarizes the primary characteristics of the five tested PC panels (TA series) from Laskar et al. (2007), as well as the two tested PC panels (PP series) from Marti and Meyboom (1992). In Table 1, $f_{c,P,i}$ is the initial compressive stress in concrete due to prestress. The other parameters were previously defined. The seven PC panels from Table 1 were computed using the refined RA-STM with efficient solution procedure and compared with the experimental results. In addition, the same panels were computed using the original RA-STM extended for PC membrane elements from Hsu and Mo (2010).

Fig. 5 presents the experimental and theoretical $\tau_{LT} - \gamma_{LT}$ curves for the PC panels ($\gamma_{LT}$ is the average shear strain) from Table 1. Two theoretical curves are presented, one computed from the refined RA-STM with efficient solution proposed in this study (refined RA-STM) and another one computed from the original RA-STM extended for PC membrane elements by Hsu and Mo (2010) (original RA-STM). For the PC panels from Marti and Meyboom (1992), the authors only present some experimental $\tau_{LT} = \gamma_{LT}$ points with error bars.

For the PC panels from Laskar et al. (2007) (TA1 to TA5), Fig. 5 shows that for most of the panels, except for Panel TA-1 for which some convergence problems are observed, the global behavior of the panels under shear is well captured by the refined RA-STM with efficient solution, namely the stiffness of the panel for the cracked stage, the yielding point (except for Panel TA-3 for which transverse reinforcement did not yield), the shear strength, and the strain 

### Table 1. Characteristics of PC Panels

<table>
<thead>
<tr>
<th>Panel</th>
<th>$f_{c,P}$ (MPa)</th>
<th>$\rho_l^{\text{long}}$ (%)</th>
<th>$\rho_L^{\text{long}}$ (%)</th>
<th>$\rho_T^{\text{long}}$ (%)</th>
<th>$f_p$ (MPa)</th>
<th>$f_{LT}$ (MPa)</th>
<th>$E_p$ (GPa)</th>
<th>$E_T$ (GPa)</th>
<th>$f_{LT}$ (MPa)</th>
<th>$E_S$ (GPa)</th>
<th>$\eta_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>TA-1</td>
<td>41.5</td>
<td>0.21</td>
<td>0.84</td>
<td>8.3</td>
<td>1.793</td>
<td>-</td>
<td>200</td>
<td>200</td>
<td>-</td>
<td>414</td>
<td>197</td>
</tr>
<tr>
<td>TA-2</td>
<td>41.3</td>
<td>0.19</td>
<td>0.84</td>
<td>8.3</td>
<td>1.793</td>
<td>-</td>
<td>200</td>
<td>200</td>
<td>-</td>
<td>77</td>
<td>192</td>
</tr>
<tr>
<td>TA-3</td>
<td>42.2</td>
<td>0.19</td>
<td>0.84</td>
<td>8.3</td>
<td>1.793</td>
<td>-</td>
<td>200</td>
<td>200</td>
<td>-</td>
<td>415</td>
<td>192</td>
</tr>
<tr>
<td>TA-4</td>
<td>42.5</td>
<td>0.19</td>
<td>0.59</td>
<td>5.8</td>
<td>1.793</td>
<td>-</td>
<td>200</td>
<td>200</td>
<td>-</td>
<td>77</td>
<td>192</td>
</tr>
<tr>
<td>TA-5</td>
<td>41.1</td>
<td>0.21</td>
<td>0.42</td>
<td>4.1</td>
<td>1.793</td>
<td>-</td>
<td>200</td>
<td>200</td>
<td>-</td>
<td>77</td>
<td>192</td>
</tr>
<tr>
<td>PP2</td>
<td>28.1</td>
<td>0.21</td>
<td>0.29</td>
<td>1.3</td>
<td>1.793</td>
<td>486</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>480</td>
<td>200</td>
</tr>
<tr>
<td>PP3</td>
<td>27.7</td>
<td>0.21</td>
<td>0.59</td>
<td>4.4</td>
<td>1.793</td>
<td>480</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>480</td>
<td>200</td>
</tr>
</tbody>
</table>

Note: 1 MPa = 145 psi.

$\rho_L = A_{LT}/A_c$; $\rho_{LT} = A_{LT}/A_c$; $\rho_T = A_{TS}/A_c$. 

Fig. 5. $\tau_{LT} - \gamma_{LT}$ curves for the reference PC panels (1 MPa = 145 psi)
initial shape of the postpeak curve. The refined RA-STM does not do well at capturing the transition between the uncracked and cracked stages because it neglects the concrete creep strength.

From Table 1, Panels TA-1, TA-2, and TA-4 do not comply with the minimum limit for \( \eta' \) \((0.4 < \eta' < 1.0)\), which is related with the softening coefficient. Because Eq. (12) was calibrated for \( 0.4 < \eta' < 1.0 \), it is expected that the influence of the softening effect is not accurately considered for these panels. To relate this observation with the bad results observed for Panel TA-1, some additional calculations were performed. It was found that, for Panel TA-1, for which \( \eta' \) is very small, small changes of \( \eta' \) deeply modify the length of the ascending branch of the theoretical \( \tau_{LT} - \gamma_{LT} \) curve after the yielding point. For Panels TA-2 and TA-4, the observed changes are much less significant.

Panel TA-1 has also a very small transverse reinforcement ratio, \( \rho_T \), which is much smaller than the minimum of 0.6% recommended by Pang and Hsu (1995), and 0.78% recommended by Zhang and Hsu (1998). This additional observation can also help to explain the worst results for Panel TA-1 when compared with Panels TA-2 and TA-4. Based on additional calculations, it was found that, for Panel TA-1, small changes of \( \rho_T \) deeply modify the results.

The previous observations seem to show that a range of validity for \( \eta' \) (or \( \eta \)) is also needed for PC membrane elements. The results also show that the refined RA-STM with efficient solution procedure can lead to worst results for PC panels when the reinforcement ratio is very small. To support the previous statements, Fig. 5 shows that for Panels TA-3 and TA-5, which all comply with the previous limits for \( \eta' \) and \( \rho \) defined for RC panels, the results are much better. The results for Panels TA-2 and TA-4, which do not comply with the minimum limit for \( \eta' \), are quite acceptable. This could indicate that for Panel PC panels, the minimum limit previously referred for \( \eta' \) can be somewhat too high.

From Fig. 5, it is also shown that the theoretical \( \tau_{LT} - \gamma_{LT} \) curves (refined RA-STM) end prematurely when compared to the experimental ones for the PC panels from Laskar et al. (2007). This is because of the assumed criteria to stop the calculation procedure, as explained previously.

For the PC panels from Marti and Meyboom (1992), Fig. 5 shows that, for the cracked stage and for low loading level, the refined RA-STM seems to agree well with the experimental results. However, it shows some difficulty to capture the final part of the \( \tau_{LT} - \gamma_{LT} \) graphs. In the experimental curve, the shear stress continues to increase for a larger deformation range and no descending branch is apparently observed. Despite this, the maximum shear, both theoretical and experimental, are not very far apart. As previously mentioned, such panels were tested without a servocontrolled system, which can explain the differences previously observed. In fact, the refined RA-STM is more suitable for modeling the behavior of panels tested under strain-control conditions. For this reason, the refined RA-STM is not likely to be able to adequately model the PC panels from Marti and Meyboom (1992) in the ultimate stage. Moreover, the height of the error bars showed in Fig. 5 shows that the variability of the experimental results is somewhat high, primarily for the ultimate stage. Again, this is probably related to the conditions under which the PC panels were tested. From the aforementioned, the results from Marti and Meyboom (1992) are considered less trustworthy to validate the proposed refined RA-STM.

From Fig. 5 it also can be stated that the predictions from the refined RA-STM with efficient solution are much better when compared to the same ones from the original RA-STM, for which the shear strength of the PC panels is noticeably overestimated. This is because of some of the refinements introduced in the refined RA-STM, which are related with the new constitutive relationships for the materials. After some simulations, it was found that the differences between the results from the refined and original RA-STM are primarily due to the \( \sigma - \varepsilon \) constitutive relationship for the non-prestress steel bars in tension. In fact, the original RA-STM incorporates a simple bilinear \( \sigma - \varepsilon \) relationship from uniaxial tensile test, while the refined RA-STM incorporates a more realistic average \( \sigma - \varepsilon \) relationship accounting for the tension stiffening effect, as previously described. To demonstrate this, Fig. 5 includes, as an example, an additional graph for Panel TA-5 [TA-5 (additional)] with the experimental and theoretical \( \tau_{LT} - \gamma_{LT} \) curves from the refined RA-STM, and also another theoretical \( \tau_{LT} - \gamma_{LT} \) curve from the modified original RA-STM (mod. original RA-STM) incorporating the average \( \sigma - \varepsilon \) relationship instead of the bilinear \( \sigma - \varepsilon \) relationship for the non-prestress steel bars in tension. The results show that the theoretical curves are coincident, except for the final part. Such a small difference is attributable to the slight differences in the average \( \sigma - \varepsilon \) relationship for the concrete in compression incorporated in each model. The refined RA-STM seems to better capture the ending of the \( \tau_{LT} - \gamma_{LT} \) curve when compared with the experimental one.

Table 2 presents, for each reference PC panel, the experimental and theoretical (from the refined RA-STM) values for the shear strength (maximum value), \( \tau_{u,exp} \) and \( \tau_{u,th} \). The ratios of the experimental to the theoretical values, \( \tau_{u,exp}/\tau_{u,th} \), are also presented. For the PC panels from Marti and Meyboom (1992), the value for \( \tau_{u,exp} \) corresponds to the average of the presented error bar. Except for Panels TA-1, PP2, and PP3, due to the aforementioned reasons, for all other PC panels from the TA series, including Panels TA-2 and TA-4, Table 2 shows that the theoretical model does well at predicting the ultimate shear stress, with an average error equal to 1.05 and a coefficient of variation equal to 7.66% for \( \tau_{u,exp}/\tau_{u,th} \). Even with all PC panels included, the results show an average value equal to 1.11 and a coefficient of variation equal to 8.86% for \( \tau_{u,exp}/\tau_{u,th} \), which are not bad. In general, the predictions from the refined RA-STM are also conservative.

Fig. 6 presents the \( \sigma_D - \varepsilon_D \) curves, both experimental and theoretical, for some PC panels from TA series as examples. Fig. 6 also include two theoretical curves, one computed from the refined RA-STM with efficient solution proposed in this study (refined RA-STM) and another one computed from the original RA-STM extended for PC membrane elements by Hsu and Mo (2010) (original RA-STM). Fig. 6 shows that the theoretical curves from the refined RA-STM agree well with the experimental ones. The refined RA-STM with efficient solution procedure better captures the behavior of concrete in compression, including the peak stress and also the postpeak behavior. Fig. 6 also shows that the original RA-STM overestimates the principal compressive stress in concrete.

Figs. 7 and 8 present, respectively, the \( \tau_{LT} - \varepsilon_{L} \) and \( \tau_{LT} - \varepsilon_{D} \) curves, both experimental and theoretical, for some PC panels from
TA series as examples. Again, it is shown that the refined RA-STM better captures the experimental behavior. The ultimate theoretical value for $\varepsilon_R$ can be much higher because the theoretical model neglects the tensile concrete strength.

By using the SMM-PC, Laskar et al. (2007) obtained good results for the same PC panels from TA series analyzed in this study, including for low loading stages. This is because this model incorporates additional refinements, such as the Poisson's effect for the whole panel in the cracked stage and the concrete tensile strength. As referred to in the “Introduction,” the solution procedure of the SMM-PC is based on a trial-and-error technique and requires a large calculation effort. The refined RA-STM with the efficient solution procedure proposed in this study, which is simpler to implement and requires less calculation effort, also

Fig. 6. $\sigma_D - \varepsilon_D$ curves for reference PC Panels TA-4 and TA-5 (1 MPa = 145 psi)

Fig. 7. $\tau_{LT} - \varepsilon_R$ curves for reference PC Panels TA-4 and TA-5 (1 MPa = 145 psi)

Fig. 8. $\tau_{LT} - \varepsilon_D$ curves for reference PC Panels TA-3 and TA-5 (1 MPa = 145 psi)
proved to be a good model to predict the ultimate behavior of PC panels under shear, namely the shear strength, which constitutes an important parameter for design.

Conclusions

In this article, a refined RA-STM with efficient solution procedure was presented to compute the behavior of PC membrane elements. From the results obtained throughout this study, the following conclusions can be drawn:

- The refined RA-STM with efficient solution procedure seems to constitute an alternative and viable theoretical model to capture the global response of PC membrane elements under shear, primarily for the cracked and ultimate stage;
- When compared with the predictions from the original RA-STM extended to PC membrane elements, the predictions from the refined RA-STM with efficient solution are found to be much better. This observation validates the refinements introduced in the RA-STM and related to the constitutive relationships for the materials;
- The refined RA-STM with efficient solution procedure is also simple to implement, requires much less calculation effort, and provides better stability when compared with the original RA-STM with solution procedure based on trial-and-error technique; and
- From the comparative analysis with experimental results, it was observed that, as for similar RC members, PC membrane elements need to comply with limits for some parameters, namely the ratio of the transverse to the longitudinal resistance force in the reinforcements and also the reinforcement ratio. The range for these parameters previously stated for RC membrane elements seems to be adequate, although the minimum limit for the first one seems to be somewhat too high.

The refined RA-STM with efficient solution procedure was checked with very few experimental results related with PC panels under shear (as Laskar et al. 2007 also did with the SMM-PC). For this reason, most of the previous conclusions should be accepted with some reserve and need to be confirmed in the future. For this, additional experimental results with PC panels under in-plane stresses are of great need.

Notation

The following symbols are used in this paper:

\[ f_{LP,i} = \text{initial tensile stress in the longitudinal prestress reinforcement due to prestress; } \]
\[ f_{LP0.1\%} = \text{proportional conventional limit stress to 0.1\% for the longitudinal prestressed steel; } \]
\[ f_{L} = \text{yielding stress of the longitudinal non-prestress reinforcement; } \]
\[ f_{P} = \text{tensile stress in the prestress reinforcement; } \]
\[ f_{Pr,i} = \text{initial tensile stress in the prestress reinforcement due to prestress; } \]
\[ f_{Pc} = \text{tensile strength of the prestressed steel; } \]
\[ f_{Sp} = \text{average tensile stress in the non-prestressed steel bars; } \]
\[ f_{Sp} = \text{uniaxial yielding stress of the non-prestressed steel bars; } \]
\[ f_{T} = \text{tensile stress in the transverse non-prestress reinforcement; } \]
\[ f_{TP} = \text{tensile stress in the transverse prestress reinforcement; } \]
\[ f_{TPr,i} = \text{initial tensile stress in the transverse prestress reinforcement due to prestress; } \]
\[ f_{TP0.1\%} = \text{proportional conventional limit stress to 0.1\% for the transverse prestressed steel; } \]
\[ f_{TPr} = \text{yielding stress of the transverse non-prestress reinforcement; } \]
\[ f_{T} = \text{yielding stress of the embedded non-prestressed bars; } \]
\[ m_{L} = \text{longitudinal proportionality coefficient; } \]
\[ m_{T} = \text{transverse proportionality coefficient; } \]
\[ \alpha_{L} = \text{angle of the principal tensile stresses in the PC membrane element; } \]
\[ \alpha_{T} = \text{angle of the principal compressive stresses in the PC membrane element; } \]
\[ \alpha_{P} = \text{angle of the principal compressive stresses in the concrete membrane element; } \]
\[ \lambda_{LT} = \text{average shear strain in the PC; } \]
\[ \varepsilon_{0} = \text{strain corresponding to the concrete compressive peak stress; } \]
\[ \varepsilon_{Pc} = \text{principal average compressive strain; } \]
\[ \varepsilon_{SP} = \text{principal average tensile strain; } \]
\[ \varepsilon_{SP} = \text{yielding strain of the longitudinal non-prestressed steel bars; } \]
\[ \varepsilon_{P} = \text{strain in the prestressed steel bars; } \]
\[ \varepsilon_{P0.7f_{P}} = \text{strain in the prestressed steel corresponding to 0.7f_{P}; } \]
\[ \varepsilon_{TPr} = \text{initial tensile strain in the prestressed steel bars due to prestress; } \]
\[ \varepsilon_{SA} = \text{average strain in the non-prestressed steel bars; } \]
\[ \varepsilon_{S} = \text{average strain for concrete in compression; } \]
\[ \varepsilon_{SP} = \text{initial compressive strain in the non-prestressed steel bars due to prestress; } \]
\[ \varepsilon_{S} = \text{transverse average strain; } \]
\[ \varepsilon_{TP} = \text{yielding strain of the transverse non-prestressed steel bars; } \]
\[ \gamma = \text{softening coefficient; } \]
\[ \rho_{L} = \text{longitudinal non-prestress reinforcement ratio; } \]
\[ \rho_{P} = \text{longitudinal prestress reinforcement ratio; } \]
\[ \rho_{T} = \text{transverse non-prestress reinforcement ratio; } \]
\[ \rho_{TP} = \text{transverse prestress reinforcement ratio; } \]
\[ \sigma_{P} = \text{principal tensile stress in the PC membrane element; } \]
\[ \sigma_2 = \text{principal compressive stress in the PC membrane element} \]
\[ \sigma_D = \text{principal compressive stress in the concrete membrane element} \]
\[ \sigma_L = \text{longitudinal normal stress in the PC membrane element} \]
\[ \sigma_L' = \text{longitudinal normal stress in the concrete membrane element} \]
\[ \sigma_T = \text{principal tensile strain in the concrete membrane element} \]
\[ \sigma_T' = \text{transverse normal stress in the PC membrane element} \]
\[ \tau_{LT} = \text{shear stress in the PC membrane element} \]
\[ \tau_T' = \text{shear stress in the concrete membrane element} \]

References


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